

Gravity Focusing of Swarms of Potential Impactors. Charles J. Byrne, Image Again, 39 Brandywine Way, Middletown, NJ 07748, charles.byrne@verizon.net.

Introduction: A meteor shower is caused by a coherent swarm of objects, potential impactors, that cross Earth's orbit about the Sun, approaching Earth from an apparent direction called the radiant. The radiant represents the vector sum of the velocity of the swarm as it encounters the Earth and the velocity of Earth at that point. Those members of the swarm that encounter Earth's atmosphere produce the spectacular flash of meteors. But what happens to those objects that miss Earth and its atmosphere entirely? Their paths are deflected by Earth into a focussed line, directly opposite to the radiant line.

Assumptions: The following analysis considers the encounter between a swarm of objects (potential impactors) such as the debris from a comet with a target object such as Earth or any other large body. It is assumed that the objects have escape velocity relative to the target, so that they follow hyperbolic trajectories relative to the target's motion. The swarm of objects is assumed to have a uniform density over a volume much larger than that of the target, and an approach velocity that is nearly uniform. Small variations in the angle of the approach velocity are characterized as a dispersion angle.

Focusing: The line that is parallel to the relative velocity vector and that passes through the center of the target is called here the radiant line (from the viewpoint of the target, this line points to the radiant in the star field). Objects that are initially near the radiant line will of course impact the target. Objects that are further from that line will pass around the target in their hyperbolic trajectories. The approaching objects that are initially in each thin cylinder at a sufficient distance D from the radiant line will pass around the target to cross an axis of focus directly opposite to the radiant line. All objects in each thin cylinder will cross the axis of focus in a small range of the distance R from the target (R increases with increasing D). As a result, the density of objects that pass through this line will be enhanced from the initial density. Further, their kinetic energy will be increased by the potential energy of the target's gravity field. This focusing effect is analogous to the light from distant quasars being focused by the gravity field of a cluster of galaxies.

The equations that describe the density enhancement and velocity of potential impactors along the line of focus are presented in general form for an arbitrary target. The parameters of Earth's gravity are used to illustrate the case of swarms of Earth-crossing objects (see Figure 1).

Fundamental equations: Equations based on the principles of conservation of energy, conservation of angular momentum, and knowledge that the potential impactors will follow conic sections relative to their target are sufficient to define their behavior [1].

$$1) \text{ total energy} = \text{kinetic energy} + \text{potential energy} \\ = \frac{1}{2}mv^2 - \mu m / r$$

where m = the mass of an object in grams, v is the velocity of the object relative to the target in km/sec, r is the radial distance of the object from the target in km, and μ is the gravitational constant of the target (for Earth, $398,603 \text{ km}^3/\text{sec}^2$).

Potential energy is zero when r is infinite and becomes negative as a body approaches the target, whose gravity field provides an increase in kinetic energy. A body must have positive total energy to escape Earth or when approaching Earth from a solar or extra-solar orbit.

Simplification of equation 1 leads to an invariant:

$$2) \quad v^2 - 2\mu / r = v_i^2$$

where v_i is the initial scalar velocity of the objects of a swarm as they approach the target (r is initially infinity).

The conservation of angular momentum (about the target as a center) provides another invariant:

$$3) \quad \text{angular momentum per mass} = vr \cos \theta$$

where θ is the angle between the velocity vector of an object and a right angle to the radius from the center of the target. A special case of angular momentum is the value at closest approach to the target, the periapse (if the target is Earth, the periapse is called the perigee). At periapse, the velocity vector is exactly at a right angle to the radius vector, so $\theta = 0$:

$$4) \quad vr \cos \theta = v_p r_p$$

where v_p is the velocity at periapse and r_p is the radius at periapse.

From equation 2), v_p can be found if v_i and r_p are known:

$$5) \quad v_p^2 = v_i^2 + 2\mu / r_p$$

The hyperbolas followed by the objects will describe conic sections about the target:

$$6) \quad 1 + e \cos \theta = h^2 / \mu r = (v_p r_p)^2 / \mu r$$

where h is the angular momentum of an object relative to the target and e is the eccentricity of the conic

section (the eccentricity is greater than 1 for a potential impactor following an hyperbola).

Set r to infinity in 6) to find the angle between the asymptote to the hyperbolic orbit and the radial to the periaipse:

$$7) \quad \text{Cos } \psi = -1/e$$

At periaipse, $\text{Cos } \theta = 1$, so the eccentricity of the orbit can be found from 6) when the velocity and radius at periaipse are known:

$$8) \quad e = (v_p^2 r_p / \mu) - 1 = (v_i^2 r_p / \mu) + 1$$

Then rotate coordinates so that the approaching asymptote (the initial velocity vector) parallel to the radiance line, becomes the horizontal axis. Solving equation 6) for r yields:

$$9) \quad r = (v_p r_p)^2 / (\mu(1 + e \text{Cos}(\phi + (\pi - \psi))))$$

This is the equation of the trajectory of a potential impactor, in circular coordinates (r, ϕ) with the origin at the target's center of gravity and the axis of focus as the axis of ϕ . We are now in a position to plot a set of trajectories of objects approaching a target at a relative velocity v_i by systematically varying r_p and solving 5), 7) and 8) for v_p, ψ , and e and then inserting their values into 9). Figure 1 shows an example of such trajectories.

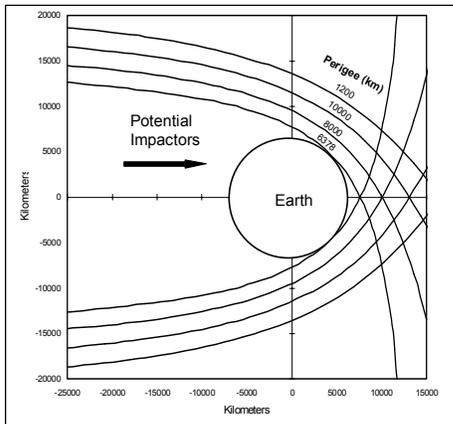


Figure 1: Trajectories of potential impactors approaching Earth with an initial velocity of 5 km/sec. The coordinates of 9) have been converted to rectangular coordinates. The density enhancement for 0.1degree dispersion at 10,000 km from the center of Earth is 25.9. The highest velocity at the focus line is 12.2 km/sec.

Density and Velocity Enhancement: All the mass initially in an annular cylinder at distance D from the line of the radiant passes through the axis of focus at radius R from the target. In a real case, the initial velocity vectors are slightly dispersed, causing

dispersal in the angles of the radiant line and the angle of the axis of focus. Consequently the focussed objects are distributed in a cone determined by the angle of dispersion. The density enhancement (ρ / ρ_i) is proportional to the cross section area of an incremental annular cylinder of objects, $2\pi D \cdot dD$, divided by dR/dD to account for the range of R covered, and also divided by the area of dispersal, $\pi(R \text{Sin}(\delta))^2$ where δ is the angle of dispersal.

To find the relation between R and D , note that the initial angular momentum (per unit mass) of an approaching object is $D v_i$. Then, by 3)

$$10) \quad D = (v_p r_p) / v_i$$

From 9) and 7), noting that $\phi = 0$ at the axis of focus:

$$11) \quad R = \frac{1}{2}(v_p r_p)^2 / \mu$$

Solving 11) for $v_p r_p$, substituting in 10) and solving for R ,

$$12) \quad R = \frac{1}{2} v_i^2 D^2 / \mu$$

$$13) \quad dR / dD = v_i^2 D / \mu$$

$$14) \quad \rho / \rho_i = 2\pi D / [(dR / dD)\pi(R \text{Sin}(\delta))^2] = 2\mu / (v_i R \text{Sin}(\delta))^2$$

The velocity of the objects as they cross the line of focus can be found from 2):

$$15) \quad v = (v_i^2 + 2\mu / R)^{1/2}$$

Discussion: Although this analysis indicates that a swarm would be deflected after one passage by a target, an actual swarm might be coalesced once more by the mutual attraction of its objects, or may be enriched by further additions, from a source comet for example.

An interesting experiment would be to examine the night sky opposite to the radiant of a predicted shower of meteors to see if the focused objects could be seen as a glow, once they pass beyond the shadow of Earth.

A satellite of a target body such as Earth may encounter objects of relatively high density and kinetic energy if it passes through a line of focus.

Reference: [1] Sonnabend, D., 1982, *Kepler, Newton, and Other Anomalous Eccentrics*, Lecture Notes